PHYSICAL REVIEW VOLUME 136, NUMBER 1B 12 OCTOBER 1964

Axial-Vector Current in Broken Unitary Symmetry*

RIAZUDDIN
Department of Physics and Astronomy, University of Rochester, Rochester, New York
(Received 22 May 1964)

By using Goldberger-Treiman-type relations and the octet version of broken unitary symmetry with the symmetry-breaking interaction behaving as T_3^3 or λ_8, Cabibbo's assumptions regarding the axial-vector interaction for leptonic decays of strongly interacting particles are justified.

RECENTLY, Cabibbo[1] proposed a theory of leptonic decays of strongly interacting particles based on the octet version of unitary symmetry.[2,3] He takes the various weak currents to be members of an octet. In his theory, vector currents for both $\Delta S=0$ and $\Delta S=1$ processes are of the F type since they are conserved in the limit of exact unitary symmetry. Let us denote by G_V and G_V' the weak coupling constants associated with $\Delta S=0$ and $\Delta S=1$ vector currents $j_\mu{}^V$ and $g_\mu{}^V$, respectively. From the decay rates of π_{e3} (known from the conserved vector-current theory of $\Delta S=0$ processes) and K_{e3}, one determines the ratio $|G_V'/G_V|$ to be $\frac{1}{4}$. Cabibbo puts $G_V=G\cos\theta$, $G_V'=G\sin\theta$, where G is the muon-decay coupling constant; then $\theta=\pm 0.25$ and one gets $G_V=0.966\ G$, which is a bit smaller than the experimental value $G_V=0.98\ G$. Sakurai[4] has given some arguments for reducing it to the experimental value. A dynamical model, which takes account of violation of the conservation of $j_\mu{}^V$ in the presence of electromagnetism and that of $g_\mu{}^V$ in the presence of mass differences such as $m_K{}^2-m_\pi{}^2$, $m_\Lambda-m_N$ etc., was considered by the author.[5,6] In the model, the divergence[5] of $j_\mu{}^V$ is associated with a $T=1$ scalar meson and that[6] of $g_\mu{}^V$ with a $T=\frac{1}{2}$ scalar meson K' of strangeness 1. It was shown how the required reduction factors come about for G_V and G_V'. For K_{l3}, the model[6] made a prediction about $\xi(=f_-/f_+)$, the ratio of the two form factors which appear in K_{l3} decay. This prediction can be tested by measuring the longitudinal polarization of the muon in $K_{\mu3}$ decay. Matthews and Salam[7] have even tried to find a representation for the vector current in which they could obtain Cabibbo's angle $\theta=-0.25$. Thus, whatever point of view one may take, the vector part of Cabibbo's theory appears to be on a reasonably secure footing.

As pointed out by Sakurai,[4] the axial-vector interaction appears to be on somewhat less secure ground in Cabibbo's theory. The reason is that while we have exact conservation of the vector current in the exact unitary symmetry limit, which reduces the number of parameters appearing in the vector interaction, the same is not true for the axial-vector current. In fact, for the axial-vector case, one has to deal with four types of currents, namely, $j_\mu{}^A(D)$, $j_\mu{}^A(F)$, $g_\mu{}^A(D)$, and $g_\mu{}^A(F)$, where $j_\mu{}^A(D)$ and $j_\mu{}^A(F)$ are D- and F-type axial-vector currents for $\Delta S=0$ processes, while $g_\mu{}^A(D)$ and $g_\mu{}^A(F)$ are the corresponding currents for $\Delta S=1$ processes. However, if one assumes that the axial-vector currents $j_\mu{}^A$ and $g_\mu{}^A$ are partially conserved in the sense that the divergences of $j_\mu{}^A$ and $g_\mu{}^A$ are proportional to the pion and the kaon fields, respectively, then Goldberger-Trieman (G-T) type relations can be used to help us fix some parameters appearing in the axial-vector interaction. G-T relations relate weak axial-vector coupling constants with strong pseudoscalar meson-baryon coupling constants so that one can exploit one's knowledge about strong interaction to deduce information about weak axial-vector coupling constants. The purpose of this paper is to explore how far one can push this program within the framework of the unitary symmetry model.

We start by considering the processes $\pi\to\mu+\nu$ and $K\to\mu+\nu$ for which the matrix elements are given by

$$G\langle 0|j_\mu{}^A|\pi\rangle[\bar\nu\gamma_\mu(1+\gamma_5)\mu]$$

and

$$G\langle 0|g_\mu{}^A|k\rangle[\bar\nu\gamma_\mu(1+\gamma_5)\mu],$$

where

$$\langle 0|j_\mu{}^A|\pi\rangle=f_\pi k_\mu,\qquad(1)$$
$$\langle 0|g_\mu{}^A|k\rangle=f_K k_\mu,$$

with $k^2=-m_\pi{}^2$ for the pion case and $k^2=-m_K{}^2$ for the kaon case. From the experimental decay rates of $\pi\to\mu+\nu$ and $K\to\mu+\nu$, one gets

$$|f_\pi/f_K|\approx m_K/m_\pi.\qquad(2)$$

If now one uses the octet version of unitary symmetry and associates with $j_\mu{}^A(D)$, $j_\mu{}^A(F)$, $g_\mu{}^A(D)$, and $g_\mu{}^A(F)$ the four coupling constants $G_A{}^D$, $G_A{}^F$, $G_A{}'^D$, $G_A{}'^F$, respectively, the axial-vector coupling constants appearing in the processes $n\to p+e+\bar\nu$, $\Lambda\to p+e+\bar\nu$, etc. are summarized in Table I. The G-T relations connecting f_π and $G_A{}^D$ and $G_A{}^F$ through the strong pion-baryon coupling constants and those connecting f_K and $G_A{}'^D$ and $G_A{}'^F$ through the kaon-baryon coupling constants are also summarized in Table I for the various processes.

* This work was supported by the U. S. Atomic Energy Commission.
[1] N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).
[2] Y. Ne'eman, Nucl. Phys. 26, 222 (1961).
[3] M. Gell-Mann, Phys. Rev. 125, 1067 (1962).
[4] J. J. Sakurai, Phys. Rev. Letters 12, 79 (1964).
[5] Riazuddin, Phys. Rev. 134, B235 (1964).
[6] Riazuddin, Nuovo Cimento 32, 1122 (1964). In this reference and in Ref. 14, K' was denoted by κ.
[7] P. T. Matthews and A. Salam, Phys. Letters 8, 357 (1964).

TABLE I. Axial-vector coupling constants and G-T relations for various leptonic processes.

Process	Axial-vector coupling constant	G-T relation
$n \to pe\bar{\nu}$	$(G_A{}^D+G_A{}^F)=G_A=1.25G_V$	$Gf_\pi=\frac{2m_N}{g_{\bar{n}p\pi^-}}(G_A{}^D+G_A{}^F)$
$\Sigma^- \to \Lambda e\bar{\nu}$	$(\frac{2}{3})^{1/2}G_A{}^D$	$Gf_\pi=\frac{(m_\Lambda+m_\Sigma)}{g_{\bar{\Sigma}^+\Lambda\pi^-}}(\frac{2}{3})^{1/2}G_A{}^D$
$\Sigma^- \to \Sigma^0 e\bar{\nu}$	$2^{1/2}G_A{}^F$	$Gf_\pi=\frac{2m_\Sigma}{g_{\bar{\Sigma}^+\Sigma^0\pi^-}}2^{1/2}G_A{}^F$
$\Xi^- \to \Xi^0 e\bar{\nu}$	$(G_A{}^D-G_A{}^F)$	$Gf_\pi=\frac{2m_\Xi}{g_{\bar{\Xi}^+\Xi^0\pi^-}}(G_A{}^D-G_A{}^F)$
$\Lambda \to pe\bar{\nu}$	$-(\frac{3}{2})^{1/2}(\frac{1}{3}G_A{}'^D+G_A{}'^F)$	$Gf_K=-\frac{(m_\Lambda+m_N)}{g_{\bar{p}\Lambda K^+}}(\frac{3}{2})^{1/2}(\frac{1}{3}G_A{}'^D+G_A{}'^F)$
$\Sigma^- \to ne\bar{\nu}$	$(G_A{}'^D-G_A{}'^F)$	$Gf_K=\frac{(m_\Sigma+m_N)}{g_{\bar{p}\Sigma^0K^+}}\frac{1}{2^{1/2}}(G_A{}'^D-G_A{}'^F)$
$\Xi^- \to \Lambda e\bar{\nu}$	$(\frac{3}{2})^{1/2}(-\frac{1}{3}G_A{}'^D+G_A{}'^F)$	$Gf_K=\frac{(m_\Xi+m_\Lambda)}{g_{\bar{\Lambda}\Xi^-K^+}}(\frac{3}{2})^{1/2}(-\frac{1}{3}G_A{}'^D+G_A{}'^F)$
$\Xi^- \to \Sigma^0 e\bar{\nu}$	$(\frac{1}{2})^{1/2}(G_A{}'^D+G_A{}'^F)$	$Gf_K=\frac{(m_\Xi+m_\Sigma)}{g_{\bar{\Sigma}^0\Xi^-K^+}}(\frac{1}{2})^{1/2}(G_A{}'^D+G_A{}'^F)$

Let us first consider the exact unitary symmetry limit where one puts all masses equal and where strong pion-baryon and kaon-baryon coupling constants are given by the exact unitary symmetry interaction Hamiltonian:

$$H_{\rm int}=g_0{}^D\,{\rm Tr}(\bar{B}PB+\bar{B}BP)+g_0{}^F\,{\rm Tr}(\bar{B}PB-\bar{B}BP)\,, \quad (3)$$

where B and P denote the baryon and pseudoscalar meson octets, respectively. Then the observation[4,8] which one makes is that for all the G-T relations listed in Table I to be consistent with one another, the F/D ratio should be the same for both the strong and weak coupling constants and that

$$G_A{}'^D/G_A{}^D=G_A{}'^F/G_A{}^F=f_K/f_\pi\,.$$

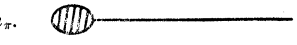
However, the point to emphasize is that in the exact unitary symmetry limit one should also put $f_\pi=f_K$ instead of $|f_\pi/f_K|=m_K/m_\pi$ as given by experiment and then one gets no useful information. This is particularly appropriate to do if one takes the point of view that the very difference between f_π and f_K may arise due to the breaking of the symmetry in the same way as the mass difference between the kaon and the pion arises. Hence, unless f_π and f_K are intrinsically different due to some obscure reason, it is more appropriate to deal with broken unitary symmetry with the symmetry-breaking interaction behaving as $T_3{}^3$ or λ_8; consideration of the latter leads to the successful Gell-Mann–Okubo (GMO) mass formula[3,9] in the octet model. Broken symmetry is also more appropriate to use for another reason, namely that in the heirarchy of interactions the mass differences are due to interactions much stronger than the weak interaction and therefore should not be neglected when dealing with the weak interaction.

For the reasons stated above, we shall now deal with broken unitary symmetry with the symmetry-breaking interaction going as $T_3{}^3$ or λ_8. First we show that for this type of broken unitary symmetry, the experimental relation (2) is perfectly plausible. Since the divergence of $j_\mu{}^A$ is proportional to the pion field, the constant f_π appearing in (1) may be defined as

$$\frac{1}{m_\pi}\langle 0|\,\partial j_\mu{}^A/\partial x_\mu\,|\pi\rangle=f_\pi m_\pi\,. \quad (4)$$

Similarly for f_K. Equation (4) may be represented by the Feynman diagram shown in Fig. 1. Now as has been discussed by Coleman and Glashow,[10] an equivalent way of taking account of the symmetry-breaking interaction $T_3{}^3$ or λ_8 for the mass differences is to consider that the mass differences arise due to an η'^0 tadpole,

FIG. 1. Feynman diagram for $f_\pi m_\pi$.



[8] N. Cabibbo, in Reports of the Conference on Fundamental Aspects of Weak Interactions, Brookhaven National Laboratories, September 1963 (unpublished).

[9] S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 949 (1962).

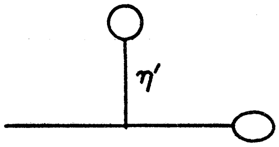


FIG. 2. Feynman diagram for the renormalization of $|f_\pi|m_\pi$ or $|f_K|m_K$ due to η'^0 tadpole.

where η'^0 is the $T=0$ member of a scalar octet. Then the insertion of the η'^0 tadpole in Fig. 1 gives rise to the Feynman diagram shown in Fig. 2. It is easy to see from Fig. 2 that if we sum over all η'^0 tadpole insertions, we obtain

$$|f_\pi|m_\pi=(k^2+m_\pi^2)[1+\delta\mu^2/(k^2+\mu_0^2)]^{-1}(k^2+\mu_0^2)^{-1}f_0\mu_0$$
$$=f_0\mu_0=|f_K|m_K,$$

where f_0 is the "bare coupling constant," μ_0 is the meson mass in the exact unitary symmetry limit, and $\delta\mu^2$ is the "self-mass" which arises due to the η'^0 tadpole.[10]

[10] S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964).

Hence $|f_\pi|m_\pi$ and $|f_K|m_K$ do not suffer any amplitude renormalization due to the η'^0 tadpole so that $|f_\pi|m_\pi/|f_K|m_K=1$ or the required relation (2). A similar argument using equal time commutators of axial-vector currents and a symmetry-breaking interaction of λ_8 type has also been given by Gell-Mann[3] to make relation (2) plausible. In a very recent paper,[11] Oehme has followed an approach similar to that of Gell-Mann[3] to obtain Cabibbo's angle for vector currents.

Having shown that the relation (2) is expected to hold for unitary symmetry broken with a $T_3{}^3$ or λ_8 interaction, we now include the symmetry-breaking effects due to $T_3{}^3$ or λ_8 in the interaction Hamiltonian (3). This we should do for the following reason: In writing down the G-T relations, one deals with two types of vertices, namely, $B\rightarrow B+P$ and $P\rightarrow l+\nu$, the former defining the pseudoscalar meson-baryon (π and K) coupling constants and the latter being determined by f_π or f_K. Now since in the G-T relations of Table I we are taking account of the mass differences between the baryons as well as between the pion and the kaon due to the symmetry-breaking interaction $T_3{}^3$ or λ_8, we should also take account of possible differences among the pseudoscalar meson-baryon coupling constants due to $T_3{}^3$ or λ_8; otherwise with pseudoscalar meson-baryon coupling constants as given by the interaction Hamiltonain (3), we cannot satisfy the G-T relations of Table I simultaneously in the presence of mass differences. The symmetry-breaking effects due to $T_3{}^3$ or λ_8 in the interaction Hamiltonian (3) have been considered by Muraskin and Glashow,[12] who show that the resulting Hamiltonian now contains seven constants instead of two as in (3). Assuming only charge independence, there are 12 pseudoscalar coupling constants to be determined between the four baryons N, Σ, Λ, Ξ and the three mesons π, K, η. Hence one gets five sum rules among the 12 constants similar to the GMO mass-sum rules. These have been explicitly written down in Ref. 12. However, out of these five sum rules only two are independent of η coupling constants and therefore these two can be used to give information about the weak axial-vector coupling constants through the G-T relations given in Table I. The two sum rules relevant for our purpose are

$$g_{\bar{\Lambda}\Xi^-K^+}-g_{\bar{\Sigma}^+\Lambda\pi^-}+(2/6^{1/2})g_{\bar{\Xi}^+\Xi^0\pi^-}+(2/3^{1/2})g_{\bar{p}\Sigma^0K^+}$$
$$-(1/3^{1/2})g_{\bar{\Sigma}^0\Xi^-K^+}+(3^{1/2}/3)g_{\bar{\Sigma}^+\Sigma^0\pi^-}=0,$$

$$g_{\bar{p}\Lambda K^+}-g_{\bar{\Sigma}^+\Lambda\pi^-}+(2/6^{1/2})g_{\bar{n}p\pi^-}-(1/3^{1/2})g_{\bar{p}\Sigma^0K^+}$$
$$+(2/3^{1/2})g_{\bar{\Sigma}^0\Xi^-K^+}-(3^{1/2}/3)g_{\bar{\Sigma}^+\Sigma^0\pi^-}=0. \quad (5)$$

From Table I, we now insert into (5) the strong pseudoscalar coupling constants in terms of the weak axial-vector coupling constants and get, on using the GMO mass formula, the following two relations between $G_A{}^D$, $G_A{}^F$, $G_A{}'^D$, and $G_A{}'^F$:

$$(1/f_K)[G_A{}'^F(1-c/c_1)-G_A{}'^D]$$
$$=(1/f_\pi)[G_A{}^F(1-c/c_1)-G_A{}^D],$$

$$(1/f_K)[G_A{}'^F(1+c/c_1{}')+G_A{}'^D]$$
$$=(1/f_\pi)[G_A{}^F(1+c/c_1{}')+G_A{}^D],$$

where

$$c=\tfrac{1}{2}(m_\Sigma-m_\Lambda),\quad c_1=m_\Xi-\tfrac{1}{2}(m_\Sigma+m_\Lambda),$$
$$c_1{}'=\tfrac{1}{2}(m_\Sigma+m_\Lambda)-m_N.$$

Solving these two equations, we obtain

$$G_A{}'^D/G_A{}^D=G_A{}'^F/G_A{}^F=f_K/f_\pi, \quad (6)$$

where $|f_K/f_\pi|\approx m_\pi/m_K\approx 1/3.6$.

Although we have taken account of the symmetry-breaking interaction $T_3{}^3$ or λ_8 at the vertices $B\rightarrow B+P$ and $P\rightarrow l+\nu$ in writing down the G-T relations, we have still been using only four weak axial-vector coupling constants $G_A{}^D$, $G_A{}^F$, $G_A{}'^D$, $G_A{}'^F$ for describing the leptonic decays of the baryons. This may appear an apparent contradiction in our approach in the sense that whereas we have claimed that in the presence of $T_3{}^3$ interaction we have seven independent pseudoscalar meson-baryon coupling constants (which include, of course, η-baryon coupling constants also, with which we do not deal as far as Table I is concerned), one may feel that by making use of Eq. (6) and Table I, one may express the pion- and kaon-baryon coupling constants in terms of just two independent constants. But this is not the case. If we use the physical masses of the baryons as we have done, then our Table I, on making use of Eq. (6), shows that the pion- and kaon-baryon coupling constants are determined by five independent constants and not two, namely, $G_A{}^D/Gf_\pi$, $G_A{}^F/Gf_\pi$ and three of the four masses m_N, m_Λ, m_Σ, and m_Ξ. In general, of course, the leptonic decays of the baryons should be described by more than four axial-vector constants

[11] R. Oehme, Phys. Rev. Letters **12**, 550 (1964).

[12] M. Muraskin and S. L. Glashow, Phys. Rev. **132**, 482 (1963).

TABLE II. Pseudoscalar meson-baryon coupling constants.

	$g^2_{NN\pi}/4\pi$	$g^2_{\Sigma\Lambda\pi}/4\pi$	$g^2_{\Sigma\Sigma\pi}/4\pi$	$g^2_{\Xi\Xi\pi}/4\pi$	$g^2_{\Lambda NK}/4\pi$	$g^2_{\Sigma NK}/4\pi$	$g^2_{\Xi\Lambda K}/4\pi$	$g^2_{\Xi\Sigma K}/4\pi$
Baryons having their physical masses	15	13.4	11	3.3	16.8	2.2	0.95	26.7
All baryons having equal mass	15	8.9	7	1.7	14	1.7	0.56	15

G_A^D, G_A^F, $G_A'^D$, $G_A'^F$ [which reduce to two on making use of Eq. (6)] when one takes account of the symmetry-breaking interaction T_3^3. But we have confined ourselves to a simpler case of using fewer axial-vector constants and since we do not get any inconsistency, it may be sufficient to confine oneself to just above four axial-vector constants, even in the presence of T_3^3 interaction.

Combining now Eq. (6) with the vector coupling relation $|G_V'/G_V|\approx\frac{1}{4}$, we obtain

$$G_A'^D/G_A^D=G_A'^F/G_A^F\approx G_V'/G_V.$$

Thus, Cabibbo's Lagrangian has been put on a somewhat more solid footing for the axial-vector interaction by means of G-T relations and broken unitary symmetry for strong interactions which is more appropriate than exact unitary symmetry in dealing with weak interactions.

Now Eq. (6) determines only two ratios $G_A'^D/G_A^D$ and $G_A'^F/G_A^F$ and hence two more relations are needed between the four axial-vector coupling constants to determine them individually. One relation is provided by the neutron β decay, namely

$$G_A^D+G_A^F=1.25G_V.$$

The other relation may be obtained from the experimental decay rate of $\Sigma^-\to\Lambda+e+\bar{\nu}$ as Cabibbo has done. This decay involves only G_A^D and one obtains

$$0.71G_V\leqslant G_A^D\leqslant 1.18G_V, \tag{7}$$

using the experimental branching ratio of $\Sigma^-\to\Lambda+e+\bar{\nu}$ as[13] $(0.9_{-0.4}^{+0.5})\times10^{-4}$. Instead of using the experimental branching ratio for $\Sigma^-\to\Lambda+e+\bar{\nu}$, which contains a large uncertainty, one may argue as follows to derive a second relation between G_A^D and G_A^F. We assume that the K' at 725 MeV is scalar and that it is coupled to K and π through the vector K^*. Then Nambu and Sakurai[14] have shown that the coupling constant of K' with K and π is given by

$$\frac{g_{K'K\pi}^2}{4\pi}=\left(\frac{\gamma^2}{4\pi}\right)\left(\frac{m^2{}_K-m_\pi{}^2}{m_{K^*}}\right)^2, \tag{8}$$

where γ is the coupling of K^* with K and π and is determined from the width of K^* to be $\gamma^2/4\pi\approx0.7$. Consider now the processes $K'\to K+l+\nu$ and $K'\to\pi+l+\nu$ which involve only the F type of axial-vector coupling constants; the corresponding G-T relations are:

$$Gf_\pi=(G_A^F/\sqrt{2}g_{K'K\pi})(m_{K'}{}^2-m_K{}^2), \tag{9}$$

$$Gf_K=(G_A'^F/\sqrt{2}g_{K'K\pi})(m_{K'}{}^2-m_\pi{}^2). \tag{10}$$

From these relations, one obtains

$$G_A'^F/G_A^F=[(m_{K'}{}^2-m_K{}^2)/(m_K'^2-m_\pi{}^2)](f_K/f_\pi) \tag{11}$$
$$\approx(1/1.7)(f_K/f_\pi).$$

This is somewhat different from what has been found in Eq. (6). This difference may be due to our use of the same value for $g_{K'K\pi}$ in Eqs. (9) and (10); this may not be appropriate since in (9) the pion is not on the mass shell while in (10) the kaon is not on the mass shell, whereas Eq. (8) for $g_{K'K\pi}$ is obtained when both these particles are on the mass shell. In any case, the discrepancy between (6) and (11) is within the other uncertainties in the calculation. Because of the lower mass of the pion, relation (9) is expected to be more accurate than (10). If we take this point of view, then using (9), (8), $\gamma^2/4\pi\approx0.7$ and the G-T relation for neutron β decay, namely, $f_\pi=(G_A/G)(\sqrt{2}m_N/g_{\pi NN})$, we obtain

$$G_A^F/G_A\approx\tfrac{1}{3},\ \text{i.e.,}\quad G_A^F\approx0.42G_V,$$

so that $G_A^D\approx0.84\ G_V$ which lies within the range (7). This gives $G_A^F/G_A^D\approx\frac{1}{2}$. This ratio agrees with the value obtained by Ryan[15] using the experimental data on the $\Sigma^-\to\Lambda e\bar{\nu}$, $\Sigma^-\to ne\bar{\nu}$, and $\Lambda\to pe\bar{\nu}$ decays. Our F/D ratio yields a branching ratio for $\Sigma^-\to\Lambda+e+\bar{\nu}$ of about 0.7×10^{-4}, consistent with the experimental value.[12] With this value of the F/D ratio and using Eq. (6), with $|f_K/f_\pi|\approx1/3.6$, we obtain for $\Lambda\to p+e+\bar{\nu}$, the magnitude of the ratio of the axial to the vector coupling constant to be about 0.78, consistent with

[13] W. Willis, R. Adair, H. Courant *et al.*, Bull. Am. Phys. Soc. 8, 349 (1963).

[14] Y. Nambu and J. Sakurai, Phys. Rev. Letters **11**, 42 (1963).

[15] Reported by R. E. Marshak, *Proceedings of the Conference on Symmetry Principles at High Energy at Coral Gables, Florida, 1964* (University of Miami, to be published).

experiment,[16] and the branching ratio to be about 0.96×10^{-3}, also consistent with experiment.[16]

Finally, we can make use of Table I and the above determination of the axial-vector coupling constants to derive the strong meson-baryon pseudoscalar coupling constants. These are summarized in Table II.

ACKNOWLEDGMENTS

The author is deeply indebted to Professor R. E. Marshak and Dr. S. Okubo for helpful comments. He would like to thank Dr. N. Cabibbo, Dr. D. Lurie, Dr. C. Ryan, and Dr. A. Zimerman for helpful discussion. A Fulbright Travel Grant by the U. S. Educational Foundation in Pakistan is gratefully acknowledged.

[16] V. G. Lund, T. O. Binford, M. L. Good, and D. Stern, Bull. Am. Phys. Soc. 9, 460 (1964); C. Baglin, V. Brisson, A. Rousset *et al.*, Phys. Letters 6, 186 (1963).

PHYSICAL REVIEW VOLUME 136, NUMBER 1B 12 OCTOBER 1964

Selection Rules and Some Relations in W_3 Symmetry

MAHIKO SUZUKI
Department of Physics, University of Tokyo, Tokyo, Japan

(Received 22 May 1964)

In the limit of W_3 symmetry proposed by Schwinger, selection rules and some relations among partial cross sections are derived through elementary calculations. One of the main results is that $\pi+N\to\Sigma+n$ pseudoscalar mesons, $\bar{K}+N\to\Sigma+n$ pseudoscalar mesons, and $\bar{K}+N\to\Xi+n$ pseudoscalar mesons are forbidden as well as their crossed reactions. Many of the selection rules and the relations derived here are badly violated at least in low-energy region. However, they should be tested with experimental data at much higher energies than those available at present.

1. INTRODUCTION

RECENTLY, Schwinger[1] proposed $U(3)\times U(3)$ symmetry or W_3 symmetry, as he called it, as an alternative to SU(3) symmetry in the strong interactions. In this paper we derived selection rules and some other relations among partial cross sections, which are readily susceptible to experimental test. These are valid in the limit of complete symmetry.

In Schwinger's scheme there are the two fundamental triplets, one is ψ_μ with baryon number 1 and the other is V_α with baryon number 2. The baryons are constructed from $\bar{\psi}$ and V, while the pseudoscalar and the vector mesons are from ψ and $\bar{\psi}$. The nine baryons are represented as the following matrix:

$$B=\bar{\psi}^\mu V_\alpha=\begin{pmatrix} (1/\sqrt{6})\Lambda+(1/\sqrt{2})\Sigma^0+(1/\sqrt{3})Y & \Sigma^- & n \\ \Sigma^+ & (1/\sqrt{6})\Lambda-(1/\sqrt{2})\Sigma^0+(1/\sqrt{3})Y & p \\ \Xi^0 & \Xi^- & -(\frac{2}{3})^{1/2}\Lambda+(1/\sqrt{3})Y \end{pmatrix}, \tag{1.1}$$

where Y is the ninth baryon. As was pointed out by Schwinger, the Y_0^* (1405 MeV) is the promising candidate for the ninth baryon, if its spin and parity turn out to be $\frac{1}{2}^+$. It should be noted here that the row and the column refer to the indices of $\bar{\psi}$ and V, respectively, in the matrix given above.[2] The assignment of the mesons is exactly the same with that of SU(3) symmetry,

$$P=\bar{\psi}^\mu\psi_\nu=\begin{pmatrix} (1/\sqrt{6})\eta+(1/\sqrt{2})\pi^0 & \pi^- & K^0 \\ \pi^+ & (1/\sqrt{6})\eta-(1/\sqrt{2})\pi^0 & K^+ \\ \bar{K}^0 & K^- & -(\frac{2}{3})^{1/2}\eta \end{pmatrix}, \tag{1.2}$$

for the pseudoscalar mesons and the similar matrix for the vector mesons. In contrast with the case of the baryons both the row and the column are simultaneously transformed under the same U(3).

2. INVARIANT AMPLITUDES OF FOUR-LEG DIAGRAMS

Let us construct invariant forms from the field operators of two baryons and two mesons. As is easily seen, the invariant amplitudes are written as

$$\begin{aligned} M=f\,\mathrm{Tr}[\{B\bar{B}-(1/3)\,\mathrm{Tr}(B\bar{B})\}(P_1P_2+P_2P_1)] \\ +g\,\mathrm{Tr}[\{B\bar{B}-(1/3)\,\mathrm{Tr}(B\bar{B})\}(P_1P_2-P_2P_1)] \\ +e\,\mathrm{Tr}(B\bar{B})\,\mathrm{Tr}(P_1P_2). \end{aligned} \tag{2.1}$$

[1] J. Schwinger, Phys. Rev. Letters 12, 237 (1964).

[2] There is another assignment which is completely equivalent from the group-theoretical viewpoint. We can obtain it by exchanging the row and the column in Eq. (1.1). In this assignment, however, the $NN\pi$ coupling turns out to be zero in the W_3-symmetric limit.